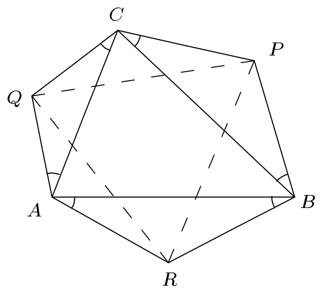
International Mathematical Talent Search – Round 4

Problem 1/4. Use each of the digits $1, 2, 3, 4, 5, 6, 7, 8, 9$ exactly twice to form distinct prime numbers whose sum is as small as possible. What must this minimal sum be? (Note: The five smallest primes are $2, 3, 5, 7$, and 11.)

Problem 2/4. Find the smallest positive integer, n, which can be expressed as the sum of distinct positive integers a, b, and c, such that $a+b$, $a+c$, and $b+c$ are perfect squares.

Problem 3/4. Prove that a positive integer can be expressed in the form $3x^2 + y^2$ if and only if it can also be expressed in the form $u^2 + uv + v^2$, where x, y, u, and v are positive integers.

Problem 4/4. Let $\triangle ABC$ be an arbitrary triangle, and construct P, Q, and R so that each of the angles marked is $30°$. Prove that $\triangle PQR$ is an equilateral triangle.



Problem 5/4. The sides of $\triangle ABC$ measure 11, 20, and 21 units. We fold it along PQ, QR, and RP, where P, Q, and R are the midpoints of its sides, until A, B, and C coincide. What is the volume of the resulting tetrahedron?