International Mathematical Talent Search – Round 37

Problem 1/37. Determine the smallest five-digit positive integer N such that $2N$ is also a five-digit integer and all ten digits from 0 to 9 are found in N and $2N$.

Problem 2/37. It was recently shown that $2^{2^{24}} + 1$ is not a prime number. Find the four rightmost digits of this number.

Problem 3/37. Determine the integers a, b, c, d, and e for which

$$(x^2 + ax + b)(x^3 + cx^2 + dx + e) = x^5 - 9x - 27.$$

Problem 4/37. A sequence of real numbers $s_0, s_1, s_2, \dots$ has the property that

$$\begin{aligned} s_i s_j &= s_{i+j} + s_{i-j} \quad \text{for all nonnegative integers } i \text{ and } j \text{ with } i \geq j, \\ s_i &= s_{i+12} \quad \text{for all nonnegative integers } i, \text{ and} \\ s_0 &> s_1 > s_2 > 0. \end{aligned}$$

Find the three numbers s_0, s_1, and s_2.

Problem 5/37. In the octahedron shown on the right, the base and top faces are equilateral triangles with sides measuring 9 and 5 units, and the lateral edges are all of length 6 units. Determine the height of the octahedron; i.e., the distance between the base and the top face.