International Mathematical Talent Search – Round 25

Problem 1/25. Assume that we have 12 rods, each 13 units long. They are to be cut into pieces measuring 3, 4, and 5 units, so that the resulting pieces can be assembled into 13 triangles of sides 3, 4, and 5 units. How should the rods be cut?

Problem 2/25. Let $f(x)$ be a polynomial with integer coefficients, and assume that $f(0) = 0$ and $f(1) = 2$. Prove that $f(7)$ is not a perfect square.

Problem 3/25. One can show that for every quadratic equation $(x - p)(x - q) = 0$ there exist constants a, b, and c, with $c \neq 0$, such that the equation $(x - a)(b - x) = c$ is equivalent to the original equation, and the faulty reasoning "either $x - a$ or $b - x$ must equal to c" yields the correct answers "$x = p$ or $x = q$".

Determine constants a, b, and c, with $c \neq 0$, so that the equation $(x-19)(x-97) = 0$ can be "solved" in such manner.

Problem 4/25. Assume that $\triangle ABC$ is a scalene triangle, with AB as its longest side. Extend AB to the point D so that B is between A and D on the line segment AD and $BD = BC$. Prove that $\angle ACD$ is obtuse.

Problem 5/25. As shown in the figure on the right, $PABCD$ is a pyramid, whose base, $ABCD$, is a rhombus with $\angle DAB = 60°$. Assume that $PC^2 = PB^2 + PD^2$. Prove that $PA = AB$.