International Mathematical Talent Search – Round 2

Problem 1/2. What is the smallest integer multiple of 9997, other than 9997 itself, which contains only odd digits?

Problem 2/2. Show that every triangle can be dissected into nine convex nondegenerate pentagons.

Problem 3/2. Prove that if x, y, and z are pairwise relatively prime positive integers, and if $\frac{1}{x} + \frac{1}{y} = \frac{1}{z}$, then $x + y$, $x - z$, and $y - z$ are perfect squares of integers.

Problem 4/2. Let a, b, c, and d be the areas of the triangular faces of a tetrahedron, and let h_a, h_b, h_c, and h_d be the corresponding altitudes of the tetrahedron. If V denotes the volume of the tetrahedron, prove that

$$(a + b + c + d)(h_a + h_b + h_c + h_d) \geq 48V.$$

Problem 5/2. Prove that there are infinitely many positive integers n such that the $n \times n \times n$ box can not be filled completely with $2 \times 2 \times 2$ and $3 \times 3 \times 3$ solid cubes.