International Mathematical Talent Search – Round 14

Problem 1/14. Let a, b, c, d be positive numbers such that $a^2+b^2+(a-b)^2 = c^2+d^2+(c-d)^2$. Prove that $a^4+b^4+(a-b)^4 = c^4+d^4+(c-d)^4$.

Problem 2/14. The price tags on three items in a store are as follows:

\$ 0.75   \$ 2.00   \$5.50

Notice that the sum of these three prices is \$8.25, and that the product of these three numbers is also 8.25. Identify four prices whose sum is \$8.25 and whose product is also 8.25.

Problem 3/14. In a group of eight mathematicians, each of them finds that there are exactly three others with whom he/she has a common area of interest. Is it possible to pair them off in such a manner that in each of the four pairs, the two mathematicians paired together have no common area of interest?

Problem 4/14. For positive integers a and b, define $a \sim b$ to mean that $ab+1$ is the square of an integer. Prove that if $a \sim b$, then there exists a positive integer c such that $a \sim c$ and $b \sim c$.

Problem 5/14. Let $\triangle ABC$ be given, extend its sides, and construct two hexagons as shown below. Compare the areas of the hexagons.