International Mathematical Talent Search – Round 13

Problem 1/13 Milo is a student at Mindbender High. After every test, he figures his cumulative average, which he always rounds to the nearest whole percent. (So 85.49 would round down to 85, but 85.50 would round up to 86.) Today he had two tests. First he got 75 in French, which dropped his average by 1 point. Then he got 83 in History, which lowered his average another 2 points. What is his average now?

Problem 2/13 Erin is devising a game and wants to select four denominations out of the available denominations \$1, \$2, \$3, \$5, \$10, \$20, \$25, and \$50 for the play money. How should he choose them so that every value from \$1 to \$120 can be obtained by using at most seven bills?

Problem 3/13 For which positive integers d is it possible to color the integers with red and blue so that no two red points are a distance d apart, and no two blue points are a distance 1 apart?

Problem 4/13 Prove that there are infinitely many ordered triples of positive integers (x, y, z) such that $x^3 + y^5 = z^7$.

Problem 5/13 Armed with just a compass — no straightedge — draw two circles that intersect at right angles; that is, construct overlapping circles in the same plane, having perpendicular tangents at the two points where they meet.