

International Mathematical Talent Search – Round 1

Problem 1/1. For every positive integer n , form the number $n/s(n)$, where $s(n)$ is the sum of the digits of n in base 10. Determine the minimum value of $n/s(n)$ in each of the following cases:

(i) $10 \leq n \leq 99$

(ii) $100 \leq n \leq 999$

(iii) $1000 \leq n \leq 9999$

(iv) $10000 \leq n \leq 99999$

Problem 2/1. Find all pairs of integers, n and k , $2 < k < n$, such that the binomial coefficients

$$\binom{n}{k-1}, \quad \binom{n}{k}, \quad \binom{n}{k+1}$$

form an increasing arithmetic series.

Problem 3/1. On an 8×8 board we place n dominoes, each covering two adjacent squares, so that no more dominoes can be placed on the remaining squares. What is the smallest value of n for which the above statement is true?

Problem 4/1. Show that an arbitrary acute triangle can be dissected by straight line segments into three parts in three different ways so that each part has a line of symmetry.

Problem 5/1. Show that it is possible to dissect an arbitrary tetrahedron into six parts by planes or portions thereof so that each of the parts has a plane of symmetry.